

A Single Classical Quark

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Abstract

The spherically symmetric solution in classical $SU(3)$ Yang - Mills theory is found. It is supposed that such solution describes a classical quark. It is regular in origin and hence the interaction between two quarks is small on the small distance. The obtained solution has the singularity on infinity. It is possible that is the reason why the free quark cannot exist. Evidently, nonlocality of this object leads to the fact that in quantum chromodynamic the difficulties arise connected with investigation of quarks interaction on large distance.

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I. INTRODUCTION

The some properties of the classical electron have the significant meaning in quantum electrodynamic. For example, an electron is the point source of electrical field, and this make possible to interpret a propagator as propagation function of point electron from initial point to endpoint. Thus, the classical properties of elementary particles have the important sense in quantum field theory.

In this work we obtain the spherically symmetric solution of the classical $SU(3)$ Yang - Mills equations. This solution we name as a “quark” (the quotes signs that some degrees of freedom are omit). This solution is regular in origin and this can means that in quantum chromodynamic the quarks weakly interact on small distance. On the other hand this solution has singularity on infinity, it is possible that this property means that a solitary quark cannot exists.

The spherically symmetric solutions for $SU(2)$ classical Yang - Mills theory had been investigated in Ref's [1]. In these papers was shown that these solutions have singularities in origin and in some distance from origin.

II. THE INITIAL EQUATIONS

The ansatz for the $SU(3)$ gauge field we take as in [2]:

$$A^a = \frac{2\varphi(r)}{Ir^2} (\lambda^2 x - \lambda^5 y + \lambda^7 z) + \frac{1}{2} \lambda^a (\lambda_{ij}^a + \lambda_{ji}^a) \frac{x^i x^j}{r^2} w(r), \quad (1a)$$

$$A_i^a = (\lambda_{ij}^a - \lambda_{ji}^a) \frac{x^j}{Ir^2} (f(r) - 1) + \lambda_{jk}^a (\epsilon_{ilj} x^k + \epsilon_{ilk} x^j) \frac{x^l}{r^3} v(r), \quad (1b)$$

here λ^a are the Gell - Mann matrixes; $a = 1, 2, \dots, 8$ is color index; Latin indexes $i, j, k, l = 1, 2, 3$ are the space indexes; $I^2 = -1$; r, θ, φ are the spherically coordinate system. Substituting Eq's (1) in the Yang - Mills equations:

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} F^{a\mu}{}_\nu) + f^{abc} F^{b\mu}{}_\nu A_\mu^c = 0, \quad (2)$$

we receive the following $SU(3)$ equations system for $f(r), v(r), w(r)$ and $\varphi(r)$ functions:

$$r^2 f'' = f^3 - f + 7fv^2 + 2vw\varphi - f(w^2 + \varphi^2), \quad (3a)$$

$$r^2 v'' = v^3 - v + 7vf^2 + 2fw\varphi - v(w^2 + \varphi^2), \quad (3b)$$

$$r^2 w'' = 6w(f^2 + v^2) - 12fv\varphi, \quad (3c)$$

$$r^2 \varphi'' = 2\varphi(f^2 + v^2) - 4fvw. \quad (3d)$$

This set of equations is very difficult even for numerical investigations. We will investigate a more simpler case when only two functions are nonzero. It is easy to see that there can be only three cases. The first case is well-known monopole case by $(f, w = 0)$ or $(v, w = 0)$. Here we will investigate only $v, w \neq 0$ case.

III. THE QUARK SOLUTION

Here we examine $f = \varphi = 0$ case. The case $v = \varphi = 0$ is analogous. Now the input equations have the following form:

$$r^2 v'' = v^3 - v - vw^2, \quad (4a)$$

$$r^2 w'' = 6wv^2. \quad (4b)$$

We seek the regular solution near $r = 0$ point. The Eq's (4) demand that v and w functions have the following view at origin $r = 0$:

$$v = 1 + v_2 \frac{r^2}{2!} + \dots, \quad (5a)$$

$$w = w_3 \frac{r^3}{3!} + \dots \quad (5b)$$

The numerical integration of Eq's (4) is displayed on Fig.1,2. The asymptotical behaviour of received solution ($r \rightarrow \infty$) is as follows:

$$v \approx a \sin(x^{1+\alpha} + \phi_0), \quad (6a)$$

$$w \approx \pm \left[(1 + \alpha)x^{1+\alpha} + \frac{\alpha \cos(2x^{1+\alpha} + 2\phi_0)}{4x^{1+\alpha}} \right], \quad (6b)$$

$$3a^2 = \alpha(\alpha + 1). \quad (6c)$$

here $x = r/r_0$ is dimensionless radius; r_0, ϕ_0 are some constants. For our potential A_μ^a we have the following nonzero color “magnetic” and “electric” fields:

$$H_\varphi^a \propto v', \quad (7a)$$

$$H_\theta^a \propto v', \quad (7b)$$

$$E_r^a \propto \frac{rw' - w}{r^2}, \quad (7c)$$

$$E_\varphi^a \propto \frac{vw}{r}, \quad (7d)$$

$$E_\theta^a \propto \frac{vw}{r}, \quad (7e)$$

$$H_r^a \propto \frac{v^2 - 1}{r^2}, \quad (7f)$$

here for Eq's (7a), (7b) and (7c) the color index $a = 1, 3, 4, 6, 8$ and for Eq's (7d), (7e) and (7f) $a = 2, 5, 7$. Analyzing the asymptotical behaviour of the $H_\varphi^a, H_\theta^a, H_r^a$ and E_φ^a, E_θ^a fields we see that they are the strongly oscillating fields. It is interesting that the radial components of the “magnetic” and “electric” fields drop to zero variously at infinity:

$$H_r^a \approx \frac{1}{r^2}, \quad (8a)$$

$$E_r^a \approx \frac{1}{r^{1-\alpha}}. \quad (8b)$$

Among all the (7) fields only the radial components of “electric” fields are nonoscillating. From Eq's (6) we see that our solution has the oscillating part (6a) and confining potential (6b). It is necessary to mark that obtained solution is solution with arbitrary initial condition, whereas the monopole solution is a special case of the initial condition. The expression for an energy density has the following view:

$$\epsilon \propto 4\frac{v'^2}{r^2} + \frac{2}{3} \left(\frac{w'}{r} - \frac{w}{r^2} \right)^2 + 4\frac{v^2 w^2}{r^4} + \frac{2}{r^2} (v^2 - 1)^2. \quad (9)$$

This function is displayed on the Fig.3.

We note the asymptotical form of the energy density followed from (7) condition:

$$\epsilon \approx \frac{2}{3} \frac{\alpha(1+\alpha)^2(3\alpha+2)}{x^{2-2\alpha}}. \quad (10)$$

In the first approximation ϵ is nonoscillating on the infinity. The asymptotical form of ϵ leads to that the energy of such solution is infinity.

IV. DISCUSSION

What is the physical meaning of this solution? It is possible that it is analogous to the Coulomb potential in electrostatics. But an electron can exist in empty space while a quark is not observable in a free state. Therefore, the obtained solution can describe the classical color charge - “quark”. The quotation marks indicate that we examine the simplified Eq’s (4) instead of complete Eq’s (3). We see that classical electron and “quark” have the fundamental difference among themselves. The electron has a singularity at origin by $r = 0$, but the “quark” at infinity by $r = \infty$. It should be also noted that this solution has the asymptotical freedom property since at origin $r = 0$ the gauge potential $A_\mu^a \rightarrow const$.

Thus, we suppose that the received here solution have the physical significance: the static spherically symmetric solution of $SU(3)$ Yang - Mills equations describes a classical single “quark”.

REFERENCES

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Fig.1. Oscillating potential $v(r)$ Fig.2. Potential $w(r)$. Fig.3. Energy density $\epsilon(r)$.





